CHAPTER ONE

SET

A set is defined as a collection of items

The Number System:

1. Our number system can be divided into the following group of set of numbers.(1) The Set of integers i.e. {……. -3,-2,-1,0,1,2,3,…… }.

– Integers refer to negative and positive whole numbers as well as zero.

2) The set of whole numbers i.e. {0, 1,2,3,4……….}.
– Whole numbers are numbers which are greater than zero, including zero itself.

3) The set of natural or counting numbers i.e. {1, 2,3,4,5 …..}.

– Natural numbers are number from 1 upwards.

4) The set of odd numbers i.e. {1, 3,5,7,9 ….}.

- Odd numbers are those numbers, which when divided by 2 always give us a remainder or a decimal, but 1 is an odd number.

5) The set of prime numbers i.e. {2, 3, 5, 7, 11, 13, 17…..}.

– Prime numbers are those numbers which have only two factors. Since 7 has two factors which are 1 and 7, then it is a prime number.

– On the other hand, 9 = 3×3 and 9 = $1\times 9 \Rightarrow 9$ has four factors, which are 3and 3, as well as 1 and 9. For this reason it is not a prime number.

6) The set of composite numbers i.e. {4, 6, 8,9,10 …..}.

– These are numbers which have two or more factors apart from itself and 1. For example apart from 1 and 20, 20 which is a composite number has four other factors which are {4 , 5} and {2, 10}.

– Also apart from 1 and 6, 6 which is a composite number has two other factors which are 2 and 3.

7) The set of even numbers i.e. {2, 4,6,8,10,12 ……}.
– These are those numbers, which can be divided by 2 without a remainder or a decimal.

8) The set of irrational numbers i.e. {$\ldots\ldots\pi, \sqrt{3},\ \sqrt{5}, \frac{1}{3}, \frac{2}{6}$, ……..}.

– This consists of square roots of numbers which does not give us whole numbers, as well as fractions without specific values. For example $\frac{1}{3}$ = 0.333333 …… and $\frac{2}{3} = 0.6666$ ….

- Lastly π or pie, even though taken to be = $^{22}/_{7}$ or 3.14, really has no fixed value.

10) Set of real numbers i.e. {…..-3,-2,-1, 0,1,2,3.5, $\sqrt{7}$…..} ,which consists of all the various sets just discussed.

FACTORS OF A GIVEN NUMBER:

- These are whole numbers which can divide that given number, without any remainder, with the given number being the highest factor. Examples are;(1). The factors of 6 = 1,2,3,6 (2) The factors of 8 = 1,2,4,8 (3) The factors of 30 = 1, 2, 3,5,6,15,30.

MULTIPLE OF A GIVEN NUMBER:

- If y is our number, then the multiples of y= $1\times y, 2\times y, 3\times y, 4\times y \dots\dots$ = y, 2y, 3y, 4y; For example, the multiples of 2= $2\times 1,\ 2\times 2, 2\times 3, 2\times 4, 2\times 5 \dots\dots\dots\dots\dots = 2, 4, 6, 8, 10 \dots\dots$Also the multiples of 5 = $5\times 1, 5\times 2, 5\times 3, 5\times 4$ …. = 5, 10, 15, 20, 25 ………………

Q1. Find the set of natural numbers from 1 to 12.

Soln

The set of natural numbers from1 to 12= {1,2,3,4,5,6,7,8,9,10,11,12} or {1,2,3,4, ... 12}.

Q2. Find the set of the even natural numbers from 1 to12.

NB: First find the set of natural numbers from 1 to 12, and select the even ones among them.

Soln

⇒{Natural numbers from 1 to 12} = {1, 2, 3 ………12}.

⇒ {Even natural numbers} = {2,4,6,8,10,12}.

Q3. Determine the set of the multiples of 3, which are less than 15.

Soln

{Multiples of 3 less than 15} = {3,6,9,12}.

Q4. Find the set of the odd multiples of 3 up to 18.

Soln

The multiples of 3 up to 18 = {3,6,9,12,15,18} and select the odd ones among them ⇒ { Odd multiples of 3 upto 18} = {3, 9, 15}.

The Number of Elements:

The number of elements of a set A is written as n(A).
Therefore if A = {a,b,c}, then n(A) = 3 and also if Y= {1,2}, then n(Y) = 2.

Types of Sets:

There are various types of sets and these are :

1. A Finite set:

- This is a set whose members can be counted, and an example is the set of people within a family.

2. An Infinite set:

- This is a set which contains an uncountable number of items or elements.

- An example is the set of the number of buckets of water that can be fetched from the sea.

3. Equal Sets:

- If A= [1,2,3} and B = {2,3,1}, then A and B are said to be equal sets.

– Two sets are said to be equal, if they contain the same elements or items, no matter the order or manner in which they have been arranged.

- Also if Z= {a,b,c,d} and X = { b,a,c,d}, then Z and X are equal sets.

4. Equivalent sets:

- These are two sets, in which the number of items or elements found in each is the same.

- For example if X = {a,b} and Y= {1,2), then X and Y are equivalent sets.

5. The Null Set:

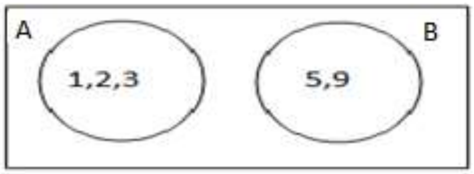
- This is a set which has no members, and it is represented by the symbol { } or Ø.

– For example {People who live in the sea} = Ø

6. Disjointed Sets:

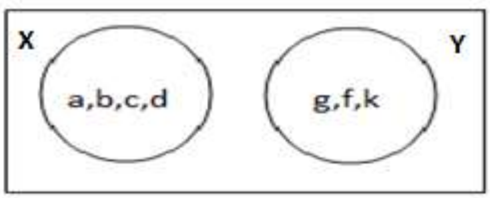
- These are two sets which do not contain any element in common.

– Example 1. If A = {1,2,3} and B = {5,9}, then A and B are disjointed sets, which can be represented on a Venn diagram as shown next:



Example 2

If X = {a,b,c,d} and Y= {g,f,k}, then X and Y are disjoined sets which can be represented on a Venn diagram as

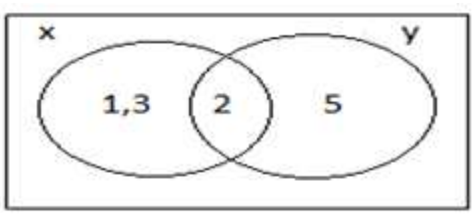


<u>7Jointed Sets</u>:

- Theses are two sets which contain one or more elements in common.

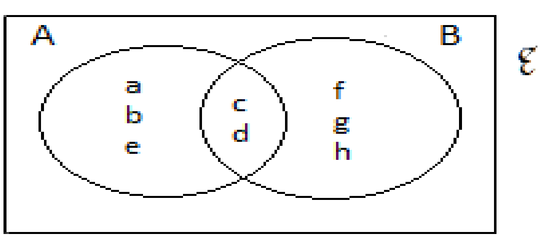
Example (1):

If x = {1, 2, 3} and Y = {2, 5}, then X and Y are jointed sets, which can be represented on a Venn diagram as:



- Example 2.

If A = {a,b,c,d,e} and B = {c,d,f,g,h}, then A and B are jointed sets, which can be illustrated on a Venn diagram as:



<u>8) The Universal Set:</u>

- This is represented by the symbol or U. It is a set which is always bigger than the set under consideration. For example if our set under consideration is {Fantis}, then any of the following sets can be Chosen as the universal set: A = {Akans}, B = {Ghanaians} and C= {Africans}.
- Also if our set under consideration is {1,2}, then any of the following sets can be chosen as the universal set: A = {1,2,3}, B = {1,2,3,4,5}, C = {Integers} and D = {Whole Numbers}.

9) Subset:

- If A = {1,2,3} and B = {2,3}, then we say that B is a subset of A, which is written as B ⊂ A or A ⊃ B. For B to be a subset of A, then(i). All the members of B must also be members of A.

(ii). The set A must contain one or more elements which are not found in B.

Example 1.

If Z = {1,2,3,4,5} and W ={2,5}, then W ⊂Z.

Example 2.
- If Y = {a,b,c,d,e} and x = {b,d,e}, then X ⊂Y.

- But if A = {1,3,7} and B = {3,8}, then B is not a subject of A, which is written as B ⊄ A or A⊅ B.

– This is due to the fact that 8 is not a member of A.
– Also if X = {1,2,3,4,5} and Y = {1,2,9,8} then Y $\not\subset X$, since 9 and 8 are not members of X.

Q1. The universal set U is given as U = {1,2,3,4,5,6,7,8,9,10} . Determine which of the following sets are subsets of the given universal set:
i. A = {1,2,3}ii. B = {5,6,10} iii. C = {8,10,44,12} iv. D = {1,8,20} v. E = {2,3,15}

NB: Before a set A can be a subset of a set B, then.

i. All the members of A must also be members of B. ii. The set B must contain one or more items, which are not found in A.

Soln

i. A = {1,2,3}. Since all the members of A are also found in the given universal set, then A is a subset of the given universal set.

ii. B = {5,6,10} }. Since all the members of B are also found in the given universal set, then B is a subset of the given universal set.

iii. C = {8,10,11,12}. Since some of the members of C {ie 11 and 12} cannot be found in the given universal set, then C is not a subset of the given universal set.

iv. D = {1,8,20}. Since 20 cannot be found in the given universal set, then D is not a subset of the given universal set.

Q2. You are given the set M = {a,b,c,d,e} . Determine which of these sets are subsets of:

i. X = {a,b,c} ii. Y= {a,e,k,m} iii. N = {c,d,e} iv. K= {a,e,g}

soln

i. X = {a,b,c}. Since all the members of the set X are also members of the set M, then X is a subset of M.

ii. Y = {a,e,k,m}. Because some of the members of Y are not members of M, then Y is not a subset of M.

iii. N = {c,d,e}. Since all the members of N are also members of M, then N is a subset of M.

(iv) K is not a subset of M, because g is not found in M but found in K.
Q3. If Q = {1,2,3,4}, write down all the possible subsets of Q.

Soln

The possible subsets are:{1,2,3}, {12},{2,4}, {2,3,4}, {4,1}, {3,1}, {3,2}, and {3, 4}.

Q4. If P = {1,2,3,4}, write down all the subsets of P having exactly two elements.

Soln

{1,2}, {13}, {1,4}, {2,3}, {2,4} and {3,4}.

The intersection:

The intersection of two sets A and B is written as $A \cap B$.

This is made up of the set of those elements, which can be found in both A and B.

Example 1)If X = {a,b,c,d} and Y= {b,c,g,h,m}, then $X \cap Y$ = {b,c}

Example 2

If A = {1,2,3,4,5,6} and B = {3,4,5,8,9,10}, then $A \cap B$ = { 3,4,5,}

Example 3If A ={1,2,3} and B = {5,6,7}, then $A \cap B$ = { }

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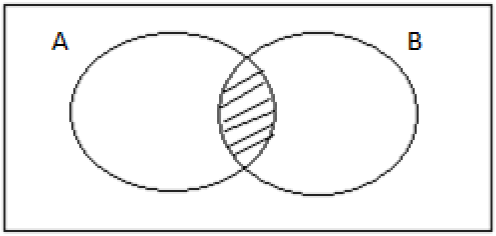
Representation of $A \cap B$ on a venn diagram:

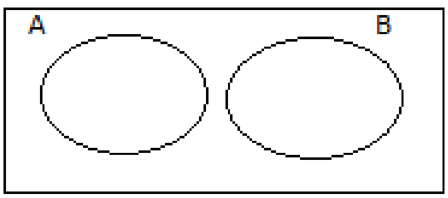


The shaded portion represents $A \cap B$.

NB:

1). If $A \cap B = \{\ \}$, i.e. if the intersection of the sets A and B is equal to the empty set, then A and B are disjointed sets.

i.e.



2).If $A \cap B \neq \varnothing$ i.e. if the intersection of A and B is not equal to the null set, then A and B are jointed sets.